4.1 Defining Momentum and Impulse

SECTION EXPECTATIONS

- Define and describe the concepts and units related to momentum and impulse.
- Analyze and describe practical applications of momentum, using the concepts of momentum.
- Identify and analyze social issues that relate to the development of safety devices for automobiles.

KEY TERMS

- momentum
- impulse
- impulse-momentum theorem

By now, you have become quite familiar with a wide variety of situations to which Newton's laws apply. Frequently, you have been cautioned to remember that when you apply Newton's second law, you must use only the forces acting on one specific object. Then, by applying Newton's laws, you can predict precisely the motion of that object. However, there are a few types of interactions for which it is difficult to determine or describe the forces acting on an object or on a group of objects. These interactions include collisions, explosions, and recoil. For these more complex scenarios, it is easier to observe the motion of the objects before and after the interaction and then analyze the interaction by using Newton's concept of a quantity of motion.

Defining Momentum

Although you have not used the mathematical expression for momentum, you probably have a qualitative sense of its meaning. For example, when you look at the photographs in Figure 4.1, you could easily list the objects in order of their momentum. Becoming familiar with the mathematical expression for momentum will help you to analyze interactions between objects.

Momentum is the product of an object's mass and its velocity, and is symbolized by $\vec{p}$. Since it is the product of a vector and a scalar, momentum is a vector quantity. The direction of the momentum is the same as the direction of the velocity.

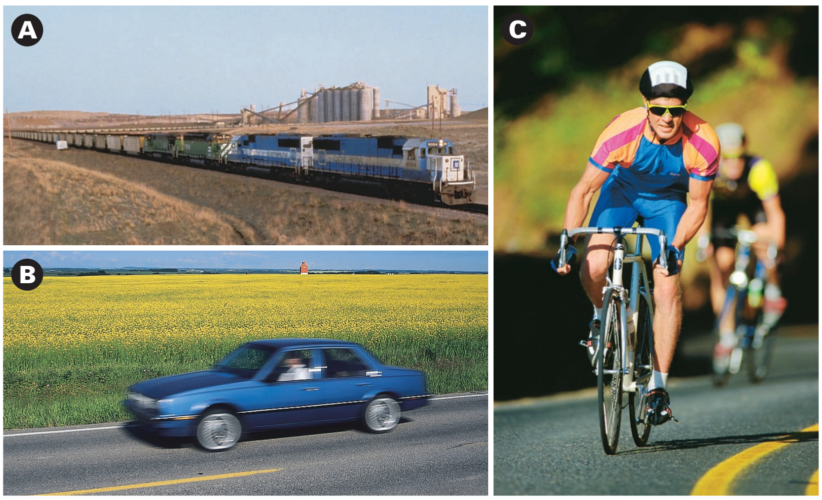


Figure 4.1 If the operator of each of these vehicles was suddenly to slam on the brakes, which vehicle would take the longest time to stop?

DEFINITION OF MOMENTUM

Momentum is the product of an object's mass and its velocity.

$$\vec{p} = m\vec{v}$$

Quantity	Symbol	SI unit
momentum	$\vec{p}$	$\frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}}$ (kilogram metres per second)
mass	m	kg (kilograms)
velocity	$\vec{v}$	$\frac{\mathrm{m}}{\mathrm{s}}$ (metres per second)

Unit Analysis

$(\text{mass})(\text{velocity}) = \mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}} = \frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}}$

Note: Momentum does not have a unique unit of its own.

SAMPLE PROBLEM

Momentum of a Hockey Puck

Determine the momentum of a 0.300 kg hockey puck travelling across the ice at a velocity of 5.55 m/s[N].

Conceptualize the Problem

- The *mass* is *moving*; therefore, it has *momentum*.
- The *direction* of an object's *momentum* is the same as the *direction* of its *velocity*.

Identify the Goal

The momentum, $\vec{p}$, of the hockey puck

Identify the Variables and Constants

Known	Unknown
$m = 0.300\ \mathrm{kg}$	$\vec{p}$
$\vec{v} = 5.55\ \frac{\mathrm{m}}{\mathrm{s}}[\mathrm{N}]$	

Develop a Strategy

Use the equation that defines momentum.

$$\vec{p} = m\vec{v}$$

$$\vec{p} = (0.300\ \mathrm{kg}) \times \left(5.55\ \frac{\mathrm{m}}{\mathrm{s}}[\mathrm{N}]\right)$$

$$\vec{p} = 1.665\ \frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}}[\mathrm{N}]$$

$$\vec{p} \cong 1.67\ \frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}}[\mathrm{N}]$$

The momentum of the hockey puck was $1.67\ \frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}}[\mathrm{N}]$.

continued ►

continued from previous page

Validate the Solution

Approximate the solution by multiplying 0.3 kg times 6 m/s. The magnitude of the momentum should be slightly less than this product, which is 1.8 kg · m/s. The value, 1.67 kg · m/s, fits the approximation very well. The direction of the momentum is always the same as the velocity of the object.

PRACTICE PROBLEM

1. Determine the momentum of the following objects.

(a) 0.250 kg baseball travelling at 46.1 m/s[E]

(b) 7.5×10^6 kg train travelling west at 125 km/h

(c) 4.00×10^5 kg jet travelling south at 755 km/h

(d) electron (9.11×10^{-31} kg) travelling north at 6.45×10^6 m/s

Defining Impulse

Originally, Newton expressed his second law by stating that the change in an object's motion (rate of change of momentum) is proportional to the force impressed on it. Expressed mathematically, his second law can be written as follows.

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

To show that this expression is fundamentally equivalent to the equation that you have learned in the past, take the following steps.

- Write the change in momentum as the difference of the final and initial momenta. $$\vec{F} = \frac{\vec{p}_\mathrm{f} - \vec{p}_\mathrm{i}}{\Delta t}$$
- Write momentum in terms of mass and velocity. $$\vec{F} = \frac{m\vec{v}_\mathrm{f} - m\vec{v}_\mathrm{i}}{\Delta t}$$
- If you assume that m is constant (that is, does not change for the duration of the time interval), you can factor out the mass, m. $$\vec{F} = \frac{m(\vec{v}_\mathrm{f} - \vec{v}_\mathrm{i})}{\Delta t}$$ $$\vec{F} = \frac{m\Delta\vec{v}}{\Delta t}$$
- Recall that the definition of average acceleration is the rate of change of velocity, and substitute an $\vec{a}$ into the above expression. $$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$ $$\vec{F} = m\vec{a}$$

Knowing that $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$ is a valid expression of Newton's second law, you can mathematically rearrange the expression to demonstrate some very useful relationships involving momentum. When you multiply both sides of the equation by the time interval, you derive a new quantity, $\vec{F}\Delta t$, called "impulse."

$$\vec{F}\Delta t = \Delta\vec{p}$$

MATH LINK

In reality, Newton expressed his second law using the calculus that he invented. The procedure involves allowing the time interval to become smaller and smaller, until it becomes "infinitesimally small." The result allows you to find the instantaneous change in momentum at each instant in time. The formulation of Newton's second law using calculus looks like this.

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Impulse is the product of the force exerted on an object and the time interval over which the force acts, and is often given the symbol $\vec{J}$. Impulse is a vector quantity, and the direction of the impulse is the same as the direction of the force that causes it.

DEFINITION OF IMPULSE

Impulse is the product of force and the time interval.

$$\vec{J} = \vec{F}\Delta t$$

Quantity	Symbol	SI unit
impulse	$\vec{J}$	N · s (newton seconds)
force	$\vec{F}$	N (newtons)
time interval	Δt	s (seconds)

Unit Analysis

(impulse) = (force)(time interval) = N · s

Note: Impulse is equal to the change in momentum, which has units of $\frac{\text{kg} \cdot \text{m}}{\text{s}}$. To show that these units are equivalent to the N · s, express N in terms of the base units.

$$\text{N} \cdot \text{s} = \frac{\text{kg} \cdot \text{m}}{\text{s}^{\cancel{2}}} \cdot \cancel{\text{s}} = \frac{\text{kg} \cdot \text{m}}{\text{s}}$$

MISCONCEPTION

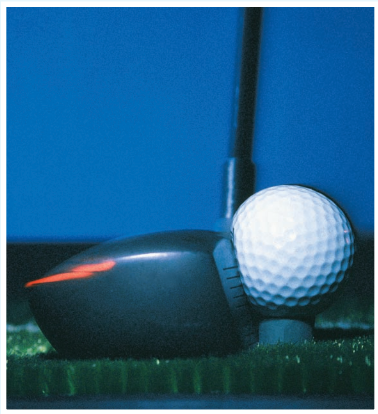
$\vec{F} = m\vec{a}$ Is Correct!

When students read the sentence "If you assume that *m* is constant (that is, does not change for the duration of the time interval), you can factor out the mass, *m*," they sometimes think that the result of the derivation, $\vec{F} = m\vec{a}$, is wrong. However, this equation is a special case of Newton's second law that is correct for all cases in which the mass, *m*, is constant. Since the mass is constant in a very large number of situations, it is acceptable to consider $\vec{F} = m\vec{a}$ as a valid statement of Newton's second law.

SAMPLE PROBLEM

Impulse on a Golf Ball

If a golf club exerts an average force of 5.25×10^3 N[W] on a golf ball over a time interval of 5.45×10^{-4} s, what is the impulse of the interaction?



Conceptualize the Problem

- The golf club exerts an *average force* on the golf ball for a period of *time*. The product of these quantities is defined as *impulse*.
- Impulse is a *vector* quantity.
- The *direction* of the impulse is the same as the *direction of its average force*.

Identify the Goal

The impulse, $\vec{J}$, of the interaction

Identify the Variables and Constants

Known	Unknown
$\vec{F} = 5.25 \times 10^3$ N[W]	$\vec{J}$
$\Delta t = 5.45 \times 10^{-4}$ s	

continued ▶

continued from previous page

Develop a Strategy

Apply the equation that defines impulse.

$$\vec{J} = \vec{F}\Delta t$$
$$\vec{J} = (5.25 \times 10^3\ \text{N[W]})(5.45 \times 10^{-4}\ \text{s})$$
$$\vec{J} = 2.8612\ \text{N} \cdot \text{s[W]}$$
$$\vec{J} \cong 2.86\ \text{N} \cdot \text{s[W]}$$

When the golf club strikes the golf ball, the impulse to drive the ball down the fairway is 2.86 N · s[W].

Validate the Solution

Round the values in the data to 5000 N[W] and 0.0006 s and do mental multiplication. The product is 3 N · s[W]. The answer, 2.86 N · s[W], is very close to the estimate.

PROBLEM TIP

When you want to use mental math to approximate an answer to validate your calculations, you can usually find the best approximation by rounding one value up and the other value down before multiplying.

PRACTICE PROBLEMS

2. A sledgehammer strikes a spike with an average force of 2125 N[down] over a time interval of 0.0205 s. Calculate the impulse of the interaction.
3. In a crash test, a car strikes a wall with an average force of 1.23×10^7 N[S] over an interval of 21.0 ms. Calculate the impulse.
4. In a crash test similar to the one described in problem 3, another car, with the same mass and velocity as the first car, experiences an impulse identical to the value you calculated in problem 3. However, the second car was designed to crumple more slowly than the first. As a result, the duration of the interaction was 57.1 ms. Determine the average force exerted on the second car.

The Impulse-Momentum Theorem

You probably noticed that the sample and practice problems above always referred to "average force" and not simply to "force." Average force must be used to calculate impulse in these short, intense interactions, because the force changes continually throughout the few milliseconds of contact of the two objects. For example, when a golf club first contacts a golf ball, the force is very small. Within milliseconds, the force is great enough to deform the ball. The ball then begins to move and return to its original shape and the force soon drops back to zero. Figure 4.2 shows how the force changes with time. You could find the impulse by determining the area under the curve of force versus time.

In many collisions, it is exceedingly difficult to make the precise measurements of force and time that you need in order to calculate the impulse. The relationship between impulse and momentum provides an alternative approach to analyzing such collisions, as well as other interactions. By analyzing the momentum before and after an interaction between two objects, you can determine the impulse.

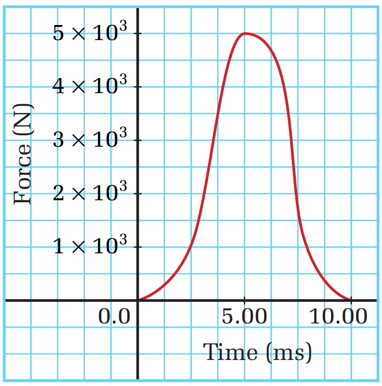


Figure 4.2 You can find the impulse of an interaction (area under the curve) by using the same mathematical methods that you used to find work done from a force-versus-position curve.

When you first rearranged the expression for Newton's second law, you focussed only on the concept of impulse, $\vec{F}\Delta t$. By taking another look at the equation $\vec{F}\Delta t = \Delta\vec{p}$, you can see that impulse is equal to the *change* in the momentum of an object. This relationship is called the **impulse-momentum theorem** and is often expressed as shown in the box below.

ELECTRONIC LEARNING PARTNER

Refer to your Electronic Learning Partner to enhance your understanding of momentum.

IMPULSE-MOMENTUM THEOREM

Impulse is the difference of the final momentum and initial momentum of an object involved in an interaction.

$$\vec{F}\Delta t = m\vec{v}_2 - m\vec{v}_1$$

Quantity	Symbol	SI unit
force	$\vec{F}$	N (newtons)
time interval	Δt	s (seconds)
mass	m	kg (kilograms)
initial velocity	$\vec{v}_1$	$\frac{\text{m}}{\text{s}}$ (metres per second)
final velocity	$\vec{v}_2$	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

(force)(time interval) = (mass)(velocity)

$\text{N}\cdot\text{s} = \text{kg}\frac{\text{m}}{\text{s}}$ $\qquad \frac{\text{kg}\cdot\text{m}}{\text{s}^{\cancel{2}}}\cancel{\text{s}} = \frac{\text{kg}\cdot\text{m}}{\text{s}}$

Note: Impulse is a vector quantity. The direction of the impulse is the same as the direction of the *change* in the momentum.

SAMPLE PROBLEM

Impulse and Average Force of a Tennis Ball

A student practises her tennis volleys by hitting a tennis ball against a wall.

(a) If the 0.060 kg ball travels 48 m/s before hitting the wall and then bounces directly backward at 35 m/s, what is the impulse of the interaction?

(b) If the duration of the interaction is 25 ms, what is the average force exerted on the ball by the wall?

Conceptualize the Problem

- The *mass* and *velocities* before and after the interaction are known, so it is possible to calculate the *momentum* before and after the interaction.
- Momentum is a *vector* quantity, so all calculations must include *directions*.

continued ►

continued from previous page

- Since the motion is all in *one dimension*, use plus and minus to denote direction. Let the *initial direction* be the *positive direction*.
- You can find the impulse from the *change* in momentum.

Identify the Goal

The impulse, $\vec{J}$, of the interaction

The average force, $\vec{F}$, on the tennis ball

> **PROBLEM TIP**
>
> Whenever you use a result from one step in a problem as data for the next step, use the unrounded form of the data.

Identify the Variables and Constants

Known		Unknown
$m = 0.060 \text{ kg}$	$\vec{v}_1 = 48 \dfrac{\text{m}}{\text{s}}$	$\vec{J}$
$\Delta t = 25 \text{ ms} = 0.025 \text{ s}$	$\vec{v}_2 = -35 \dfrac{\text{m}}{\text{s}}$	$\vec{F}$

Develop a Strategy

Use the impulse-momentum theorem to calculate the impulse.

$$\vec{F}\Delta t = m\vec{v}_2 - m\vec{v}_1$$
$$\vec{F}\Delta t = 0.060 \text{ kg}\left(-35 \frac{\text{m}}{\text{s}}\right) - 0.060 \text{ kg}\left(48 \frac{\text{m}}{\text{s}}\right)$$
$$\vec{F}\Delta t = -2.1 \frac{\text{kg}\cdot\text{m}}{\text{s}} - 2.88 \frac{\text{kg}\cdot\text{m}}{\text{s}}$$
$$\vec{F}\Delta t = -4.98 \frac{\text{kg}\cdot\text{m}}{\text{s}}$$
$$\vec{F}\Delta t \cong -5.0 \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

(a) The impulse was 5.0 kg · m/s in a direction opposite to the initial direction of the motion of the ball.

Use the definition of impulse to find the average force.

$$\vec{F}\Delta t = -4.98 \frac{\text{kg}\cdot\text{m}}{\text{s}}$$
$$\vec{F} = \frac{-4.98 \frac{\text{kg}\cdot\text{m}}{\text{s}}}{\Delta t}$$
$$\vec{F} = \frac{-4.98 \frac{\text{kg}\cdot\text{m}}{\text{s}}}{0.025 \text{ s}}$$
$$\vec{F} = -199.2 \text{ N}$$
$$\vec{F} \cong -2.0 \times 10^2 \text{ N}$$

(b) The average force of the wall on the tennis ball was 2.0×10^2 N in the direction opposite to the initial direction of the ball.

Validate the Solution

Use an alternative mathematical technique for the impulse calculation by factoring out the mass, subtracting the velocities, then multiplying to see if you get the same answer.

$$\vec{F}\Delta t = m(\vec{v}_2 - \vec{v}_1)$$
$$\vec{F}\Delta t = 0.060 \text{ kg}\left(-35\frac{\text{m}}{\text{s}} - 48\frac{\text{m}}{\text{s}}\right)$$
$$\vec{F}\Delta t = (0.060 \text{ kg})\left(-83 \frac{\text{m}}{\text{s}}\right)$$
$$\vec{F}\Delta t = -4.98 \frac{\text{kg}\cdot\text{m}}{\text{s}} \cong -5.0 \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

Check the units for the second part of the problem.

$$\frac{\frac{\text{kg}\cdot\text{m}}{\text{s}}}{\text{s}} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2} = \text{N}$$

PRACTICE PROBLEMS

5. The velocity of the serve of some professional tennis players has been clocked at 43 m/s horizontally. (Hint: Assume that any vertical motion of the ball is negligible and consider only the horizontal direction of the ball after it was struck by the racquet.) If the mass of the ball was 0.060 kg, what was the impulse of the racquet on the ball?

6. A 0.35 kg baseball is travelling at 46 m/s toward the batter. After the batter hits the ball, it is travelling 62 m/s in the opposite direction. Calculate the impulse of the bat on the ball.

7. A student dropped a 1.5 kg book from a height of 1.75 m. Determine the impulse that the floor exerted on the book when the book hit the floor.

Impulse and Auto Safety

One of the most practical and important applications of impulse is in the design of automobiles and their safety equipment. When a car hits another car or a solid wall, little can be done to reduce the change in momentum. The mass of the car certainly does not change, while the velocity changes to zero at the moment of impact. Since you cannot reduce the change in momentum, you cannot reduce the impulse. However, since impulse ($\vec{F}\Delta t$) depends on both force and time, engineers have found ways to reduce the force exerted on car occupants by extending the time interval of the interaction. Think about how the design of a car can expand the duration of a crash.

In the early days of auto manufacturing, engineers and designers thought that a very strong, solid car would be ideal. As the number of cars on the road and the speed of the cars increased, the number and seriousness of accident injuries made it clear that the very sturdy cars were not protecting car occupants. By the late 1950s and early 1960s, engineers were designing cars with very rigid passenger cells that would not collapse onto the passengers, but with less rigid "crumple zones" in the front and rear, as shown in Figure 4.3.

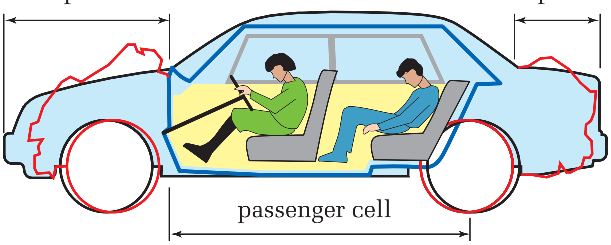


Figure 4.3 Although a car crash seems almost instantaneous, the time taken for the front or rear of the car to "crumple" is great enough to significantly reduce the average force of the impact and, therefore, the average force on the passenger cell and the passengers.

PROBEWARE

If your school has probeware equipment, visit ***www.mcgrawhill.ca/links/physics12*** and follow the links for an in-depth activity on impulse and momentum.

ELECTRONIC LEARNING PARTNER

Use the crash test provided by your Electronic Learning Partner to enhance your understanding of momentum.